MELIORATION OF SELECTION METHODS OF SYNCHRONOUS PAIRS

By

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Introduction

The tracking of satellites may be used for the determination of space position of a satellite in a coordinate system of reference station for the melioration of elements in the orbit of the satellite, for the determination of unknown coordinates of stations etc. To solve these problems; it is necessary to have synchronous pairs. However, with the utilization of passive satellites, only quasi-synchronous observations may be realistically made. The calculation of satellite coordinates corresponding to the moment of synchronization is effected by graphic or analytical methods. The author suggests the utilization an interpolated Lagrange formula. At the Institute of Theoretical Astronomy a program was compiled for the computation of the Lagrange polynomial on a BESM-2 computer. The results of these calculations are presented in this article. Synchronous tracking of satellites has received wide use during the past few years. These observations may be utilized for the determination of space location of the satellite in a coordinate system of reference stations, for the melioration of orbit elements of a satellite, for the determination of coordinates of unknown stations and for the solution of a series of other problems. When solving these problems, it is necessary to have synchronous pairs, that is, the values of the

the topocentrical dimensions and the direction to the satellite or the distance to it obtained by several stations in one moment of time.

However, with the utilization of passive satellites, that is, satellites not rendering light signals, only quasi-synchronous observations may be effected. With these observations, at every station within a certain lapse of time, the topocentric coordinates of the satellites are measured independantly in this case. As a result, in a series of moments

$$t_1, t_2, \ldots, t_n$$

we have a series of observed quantities

$$y_1, y_2, \ldots, y_n$$

Moreover, y_i characterizes either the topocentrical direction to the satellite/α, δ, h or A/, which is the distance to it /ρ/.

Since the observation moment t_i for the various stations does not usually coincide, however they may be close to one another, then a problem arises - to compute the synchronous coordinate pairs for a certain moment t, satisfying the condition $t_1 < t < t_n$.

This problem may be solved by either graphic or analytical methods, which have been suggessted by various authors [1,2,3]. The graphic method, which is the simplest in concept, initially received wide utilization. Calculating by the axis abscissa of the moment of time t_i, and by the axis of the ordinates which is the observation of dimensions y_i, and then conducting through the obtained points smooth curve, one can receive the known dimensions of with this curve y to the ecliptic moment t. The values $y^{(1)}$ and $y^{(2)}$, found in this fashion for two stations well represented in themselves, a synchronous pair of topocentric coordinates.

The advantage of the graphical method and its simplicity is contained in that for which it allows to immediately reject those quantities y_i which contain errors. The main disadvantage is that it is very time consuming and does not permit computerized calculations. With the calculation of large series of observations this method takes a considerable amount of time and effort.

This is one reason why various analytical methods were presented which permit the mechanization of the computations. The empirical formula, with assistance which all presented observations are made can be selected as a polynomial $2_m/t/c$ with a small number of parameters - A_o, $A_1, \ldots, A_m$. If the number of parameters in m = 1 are less than the quantity of observations in, then the parameters are determined by a method of lesser squares. Luthering ourselves, for example, by polynomial of the third order $Q_3/t/c$ 4- with four parameters we can express

$$Q_3/t/ = A_o + A_1/t-t_1/+A_2/t-t_1/^2+A_3/t-t_1/^3.$$

The main advantage of this method is its ease of programming the parameters A_j / j = 0, 1,..., m/ in computers. The difficulty consists in the selection of numbers of parameters. If the number of parameters is significantly less than the number of observations, then the separate observations may have systematic declinations from the obtained analytical curve 2. In this case all of the combined observations may be subdivided into several parts and for each one a determination of its own parameters is made. The example of such calculations is given in work [1].

However, another method may be used. If for moment t_i there is a series of observed quantities $y_i/i=1,2,\ldots,n$, then one can construct a function $L/t_i=y_i$, $L/t\ 2 = y_2,\ldots,$ $L/t_n/=y_n$. Function L/t will represent itself as a polynomial n-1 cautions in first degree while the value of the argument t_i are designated as a node of the interpolations. Such forms of polynomial are well-known for example, polynomials of Newton, Lagrange and others. It is expedient in this case to utilize the interpolation of the Lagrange polynomials, since the other remaining polynomials are presented for equal values of the argument. Having n value of the argument t_i and function y_i, we can express an interpolate Lagrange formula in the following manner:

$$L_{n-1}(t) = \sum_{i=1}^{n} f_i(t) y_i, \qquad /3/$$

where

$$f_i(t) = \frac{(t-t_1)(t-t_2)\ldots(t-t_{i-1})(t-t_{i+1})\ldots(t-t_n)}{(t_i-t_1)(t_i-t_2)\ldots(t_i-t_{i-1})(t_i-t_{i+1})\ldots(t_i-t_n)} \qquad /4/$$

Formula 3 permits the calculation of the values of quantity y at a given moment t. The interpolated curve, given by formulas

3 and 4 traverses through all values of y_i, and consequently, in this case there will be no systematic declinations of observations from the given curve. However, if some of the observed quantities y_i contain contingent errors, then the interpolated curve L/t will repeat these errors. Then it becomes very expedient to exclude from series I the observed quantities y_i of these values which contain the most contingent errors and to construct curve 3 utilizing only the remaining interpolation nodes. Since this problem does not have a strict or a single solution, one can attempt to resolve it by approximated methods. One of the possible methods of the solution may be as follows.

First of all, utilizing all observations y_i, we compute by the polynomial $2_m/t$ in the form of 2 that is, we determine by a method of lesser squares of its parameters $A_o, A_1, \ldots, A_m$. Usually it is sufficient tolimit ourselves with the polynomial of the 3rd or 4th degree. Then we calculate the value of polynomial Q_m/t_i for all t_i and find the difference of observed and calculated quantities

$$\mathcal{E}_i = y_i - Q_m(t_i). \qquad /5/.$$

Note, that if $\mathcal{E}_i$ significantly exceeds the accuracy of the observations, then we can either increase the degree of polynomial $2_m/t$ to the one minute or limiting ourselves for the same polynomial to vary the aggregate of the observation quantity 1 into two parts, from each one the determination of the parameters A_j

would be effected.

Obtaining value in $\mathcal{E}_i$, we calculate the average value of $\bar{\mathcal{E}}$ by formula

$$\bar{\varepsilon} = \frac{\sum_{i=1}^{n} |\varepsilon_i|}{n} \qquad /6/$$

Quantities $\mathcal{E}_i$, which are found by formula 5, characterize to a certain degree, the accuracy of each separate observation. The explured from further processing these observations for which $|\varepsilon_i| > 3\bar{\varepsilon}$. In this manner, the polynomial Q_m/t in this case plays an auxillary role and serves for the processing of erroneous observations. Examining all remaining values of arguments t_i as interpolation nodes, we construct and interpolated Lagrange curve L/t, determining formula 3 and 4. We obtained curve L/t serves as a direct determination of quantities y for selected synchronous moment t.

In the Institute of Theoretical Astronomy a program was compiled for the calculation of the Lagrange polynomials by formulas 3 and 4 on a BESM-2 computer. By the given interpolation nodes $t_1, t_2, \ldots, +n$ and by the corresponding values of functions $y_1, y_2, \ldots, y_n$ this program permits for any argument t to calculate the value of the Lagrange polynomial y = L/t.

Below are given results of investigations of two observation series. In tables 1 and 2 are presented topocentric declination data δ for satellite 65061, observed on September 21, 1965 at Stations No. 1024 - Kishenev and No. 1017 - Dneprotepousic, path No. 132 [4]. In these tables t is the universal observed station time a $\varepsilon_\delta, \varepsilon'_\delta$ - are factors calculated by formula 5. Note that the theoretical accuracy of topocentric coordinate determination from visual observations at an order of $0.^h01$ by α and δ $0.^o1$ by

For station No. 1024 - Kishinev we have;

Table 1

№ observed	t	δ	ε_δ	ε'_δ
I	$I7^h55^m2I.^s9$	+ 53°50'	0°46	0°24
2	55 27.9	55 50	- 0.07	- 0.I0
3	55 3I.9	56 50	- 0.4I	- 0.34
4	55 38.4	58 30	- 0.33	- 0.I6
5	55 42.6	59 40	0.I6	0.38
6	56 42.0	48 55	0.35	- 0.02
7	56 43.4	48 20	0.29	- 0.09
8	56 46.I	47 40	0.67	0.25
9	56 56.0	43 30	0.39	- 0.I2
I0	56 58.8	42 25	0.4I	- 0.II
II	57 0I.I	38 I0	- 2.95	-
I2	57 I0.4	38 I0	0.59	0.08
I3	I7 57 I8.9	+ 35 00	0.43	0.0I

Processing the observations on the first by the 13th and limiting the polynomial to the 3rd order, we obtained

$$\delta = 53.^\circ372 + 28.^\circ6231/t-t_1/ - 33.^\circ9400/t-t_1/^2 + 7.^\circ3427/t-t_1/^3, \quad /7/$$

where $t - t_1/$ while, a $t_1 = 17^h55^m21^s.9$.

Then by formula 6 we calculate $\bar{\varepsilon}_\delta = 0.^o58$ and find that only a 2nd point $|\varepsilon_\delta| = 2.95 > 3\bar{\varepsilon}_\delta$. This observation, obviously contains a large contingent error. If we construct a polymonial in the form of 2 by all of the aggregate observation including II, we obtain:

$$\delta = 53^o.594 + 26^o.4648/t-t_1/ - 31^o.0668/t-t_1/^2 + 6^o.4632/t-t_1/^3. \quad /8/$$

Then determining for polynomial 8, ε'_δ, and comparing them with corresponding ε_δ, we see that almost all ε'_δ are less ε_δ. Consequently polynomial 8 presents a bigger picture of aggregate observations and polynomial 7. Therefore, with the calculation of the Lagrange polynomial 3 it follows in the capacity of interpolation nodes to moments t in table 1 excluding moments $t_{11} = 17^h57_m01.^s1$.

Now we examine the satellite observations concluded in station No. 1017 at Dneprotepousic.

Table 2

No. observed	t	δ	ε_δ
I	I7ʰ56ᵐ06ˢ.6	+ 32°45'	0°.04
2	56 II.6	33 30	0.00
3	56 2I.6	34 45	- 0.09
4	56 3I.5	35 45	- 0.03
5	56 4I.I	36 I5	- 0.05
№ набл.	t	δ	ε_δ
6	I7ʰ56ᵐ47ˢ.6	+ 36°30'	+ 0°.08
7	56 48.5	36 30	0.08
8	56 50.7	36 25	0.0I
9	56 52.2	36 25	0.04
I0	57 I0.4	35 05	- 0.I2
II	I7 57 33.9	+ 3I 00	0.03

Presenting the aggregate of the observations with the aid of polynomial in the form of 2, we obtain:

$$\delta = 32^\circ\!.706 + 10^\circ\!.0981/t-t_1/ - 6^\circ\!.0090/t-t_1/^2 - 1^\circ\!.2029/t-t_1/^3, \qquad /9/$$

where $t_1 = 17^h56^m06^s\!.6$. Then we calculate for every observation ε_δ and by formula 6 we find $\bar{\varepsilon}_\delta = 0^\circ\!.05$. No observation $/\varepsilon_\delta/$ exceeded $3\bar{\varepsilon}_\delta$. Consequently it can be assumed that polynomial 9 well represents the aggregates of the observations. However, the analysis of ε_δ show that the declination of the observed quantities δ from those values which were calculated by formula 9 have a systematic character.

This is why it is expedient to present the aggregate of the table values with the aid of Lagrange polynomial 3 examining all δ values of t as interpolation nodes. Note that the suggessted methods may be utilized not only with the selection of synchronous pairs. It may be implimented in any case when, for arguments $t_1, t_2, \ldots, t_n$ are given tables of function values $y_1, y_2, \ldots, y_n$ and at this there is a necessity to calculate the value of y for argument t which satisfies the conditions $t_1 < t < t_n$.

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